① ommitted var.

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \rho v + u$$

② $y = \beta_0^* + \beta_1^* x^* + u$

$x = x^* + e \Rightarrow \boxed{x^* = x - e}$

$y = \beta_0 + \beta_1 x + \varepsilon$

ε … e

$y = \beta_0^* + \beta_1^*(x - e) + u$

$= \beta_0^* + \beta_1^* x + u - \beta_1^* e$

$$\hat{\beta}_1 = \frac{\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum(x_i - \bar{x})^2} \xrightarrow{P} \frac{cov(x,y)}{V(x)} = \frac{cov(x^* + e, \beta_0^* + \beta_1^* x^* + u)}{V(x^* + e)}$$

$$= \boxed{\beta_1^* \frac{V(x^*)}{V(x^*) + V(e)}}$$

attenuation bias

③ RDP

$$\begin{cases} y_1 = \beta_0 + \beta_1 y_2 + \beta_2 x_2 + \cdots + \beta_k x_k + u \\ y_2 = \delta_0 + \delta_1 y_1 + \delta_2 m_2 + \cdots + \beta_J m_J + e \end{cases}$$

$y = \gamma_0 + \gamma W + \delta_1 x_1 + \cdots + \delta_k x_k + u$

④ self-selection

$W = 0/1$

$y(0) = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u_1$

$y(1) = \delta_0 + \delta_1 x_1 + \cdots + \delta_k x_k + u_2$

$y(1) - y(0) = (\delta_0 - \beta_0) + (\delta_1 - \beta_1) x_1 + \cdots + (\delta_k - \beta_k) x_k + u_2 - u_1$

$= \delta_0 - \beta_0 + u_2 - u_1$

$W = 1\{y(1) > y(0)\} = 1\{\delta_0 - \beta_0 + u_2 - u_1 > 0\} \Longrightarrow$ Roy model

$y = w\,y(1) + (1-w)\,y(0)$

$= w(\delta_0 + \delta_1 x_1 + \cdots + \delta_k x_k + u_2) + (1-w)(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u_1)$

$= \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + (\delta_0 - \beta_0) w + [w u_2 + (1-w) u_1]$

$Y_i = A_i K_i^{\alpha} L_i^{\beta}$

$\ln Y_i = \ln A_i + \alpha \ln K_i + \beta \ln L_i$

max $pY - rK - wL \Rightarrow K(A)$, $L(A)$

quality $= \alpha + \beta \cdot$ quantity $+ \cdots + u$

policy

Bartik / shift-share

$e_i = \sum_m [e_{im}] S_{im}$

$\hat{e}_m$

z_i

simulated IV

结构: $y=\beta_0+\beta_1 x+u$ ⇐ 因果

$+\beta_2 m$

⇒ 1st stage: $x=\delta_0+\delta_1 z+e$ ⇐ 相关性

$+\delta_2 m$

$\hat{\delta}_1$

⇒ $y=(\beta_0+\beta_1\delta_0)+\beta_1\delta_1 z+u+\beta_1 e$

简约式: $y=\eta_0+\eta_1 z+\varepsilon$

$\hat{\eta}_1$ $\quad +\eta_2 m$

ITT

$$\hat{\eta}_1=\frac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})^2}$$

$$\hat{\delta}_1=\frac{\sum(z_i-\bar{z})(x_i-\bar{x})}{\sum(z_i-\bar{z})^2}$$

$$\eta_1=\beta_1\delta_1$$

$$\beta_1=\frac{\eta_1}{\delta_1}$$

Wald 估计

$$\hat{\beta}_1=\frac{\hat{\eta}_1}{\boxed{\hat{\delta}_1}}=\frac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})(x_i-\bar{x})}$$

$$y_1\,\Big|_z\ E\{\beta_0+\beta_1 y_2+\beta_2 z_1+\cdots+\beta_k z_{k-1}+u|z\}\Big|_z$$

$$E(y_1|z)\quad E(y_1|z_k)=\beta_0+\beta_1E(y_2|z_k)+\beta_2E(z_1|z_k)+\cdots$$

$$y_1=\beta_0+\beta_1 y_2+\beta_2 z_1+\cdots+\beta_k z_{k-1}+u_1$$

z_k

$z=\begin{pmatrix}z_1\\ \vdots\\ z_k\end{pmatrix}$ ⇒ 所有外生变量.

$$E(y_1|z)=\beta_0+\beta_1E(y_2|z)+\beta_2 z_1+\cdots+\beta_k z_{k-1}$$

$$y_1=\beta_0+\beta_1\boxed{E(y_2|z)}+\beta_2 z_1+\cdots+\beta_k z_{k-1}+\varepsilon$$

$$\varepsilon=y_1-E(y_1|z)$$

① $y_2=\pi_0+\pi_1 z_1+\cdots+\pi_k z_k+\pi_{k+1}z_{k+1}+e$

$\hat{y}_2$

② $y_1=\beta_0+\beta_1\hat{y}_2+\beta_2 z_1+\cdots+\beta_k z_{k-1}+\varepsilon$

$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 y_3 + \beta_3 z_1 + \cdots + \beta_k z_{k-2} + u_1$

z_{k-1}, z_{k+1} z_k

① $y_2 = \pi_0 + \pi_1 z_1 + \cdots + \pi_{k-2} z_{k-2} + \pi_{k-1} z_{k-1} + \pi_k z_k + \pi_{k+1} z_{k+1} + v_2$

$y_3 = \cdots$

$\hat{y}_2$

$\hat{y}_3$

reg $\hat{\varepsilon}$ on $z_1\ z_2 \cdots \cdots z_k\ z_{k+1} \cdots = F \cdots + v_3$

② $y_1 = \beta_0 + \beta_1 \hat{y}_2 + \beta_2 \hat{y}_3 + \beta_3 z_1 + \cdots + \beta_k z_{k-2} + \varepsilon$

$\hat{\varepsilon} = y_1 - \hat{\beta}_0 - \hat{\beta}_1 \hat{y}_2 - \hat{\beta}_2 \hat{y}_3 - \hat{\beta}_3 z_1 - \cdots$

$\hat{\varepsilon} = y_1 - \hat{\beta} - \hat{\beta} y_2 - \hat{\beta} y_3$

$H_0: \pi_{k-1} = \pi_k = \pi_{k+1} = 0$

1st - F-value

$H_0: \delta_{k-1} = \delta_k = \delta_{k+1} = 0$

underidentification test rk

H_0: 无内生性

H_1: 有内生性.

	OLS	IV
H_0	一致	一致
H_1	不一致	一致

Hausman检验

$\begin{pmatrix} u_1 \\ v_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \cdots \\ \rho & v_2 \end{pmatrix}\right)$

$y_1 = 1\{\beta_0 + \beta_1 y_2 + \cdots + u_1 > 0\}$

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 z_1 + \cdots + \beta_k z_{k-1} + u_1$$

1st $\rightarrow$ $y_2 = \pi_0 + \pi_1 z_1 + \cdots + \pi_k z_k + v_2$

(1) $u_1 = \rho v_2 + e$

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 z_1 + \cdots + \beta_k z_{k-1} + \rho \hat{v}_2 + e$$

$\hat{v}_2$ $\quad H_0: \rho = 0$

(2) $\begin{pmatrix} u_1 \\ v_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, (\;)\right)$

MLE

$f(y_1, y_2 \mid \cdots)$

LIML